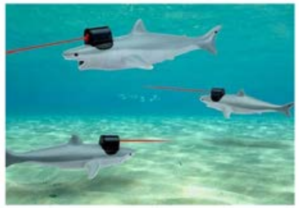
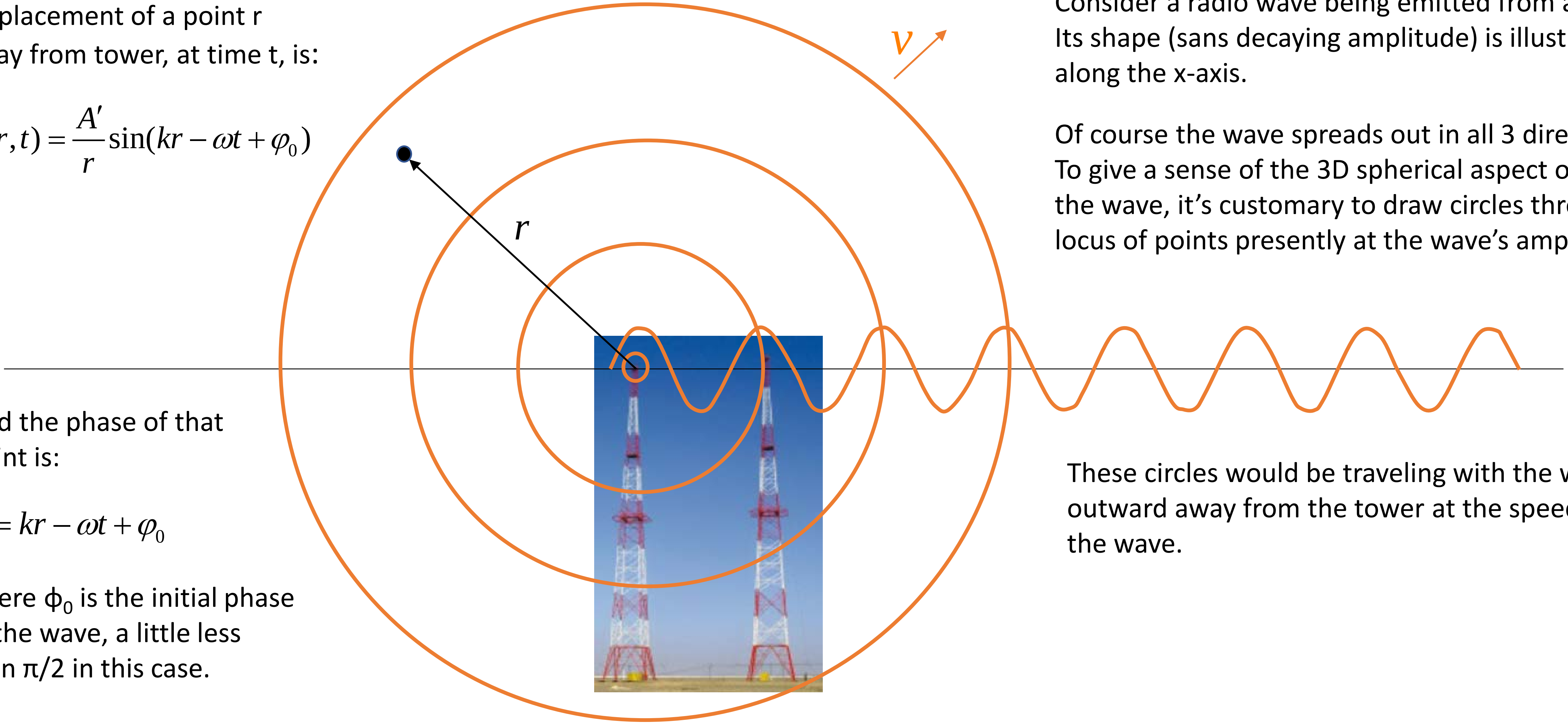
D.4 Multisource Interference



So far we've been considering interference between a wave and its reflections. Now let's consider the interference between two (or more) different waves. First some basic illustrations.

Displacement of a point r away from tower, at time t, is:

$$D(r,t) = \frac{A'}{r}\sin(kr - \omega t + \varphi_0)$$



Consider a radio wave being emitted from a tower. Its shape (sans decaying amplitude) is illustrated along the x-axis.

Of course the wave spreads out in all 3 directions. To give a sense of the 3D spherical aspect of the wave, it's customary to draw circles through the locus of points presently at the wave's amplitude.

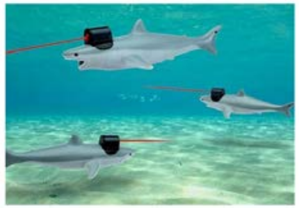
And the phase of that point is:

$$\varphi = kr - \omega t + \varphi_0$$

where ϕ_0 is the initial phase of the wave, a little less than $\pi/2$ in this case.

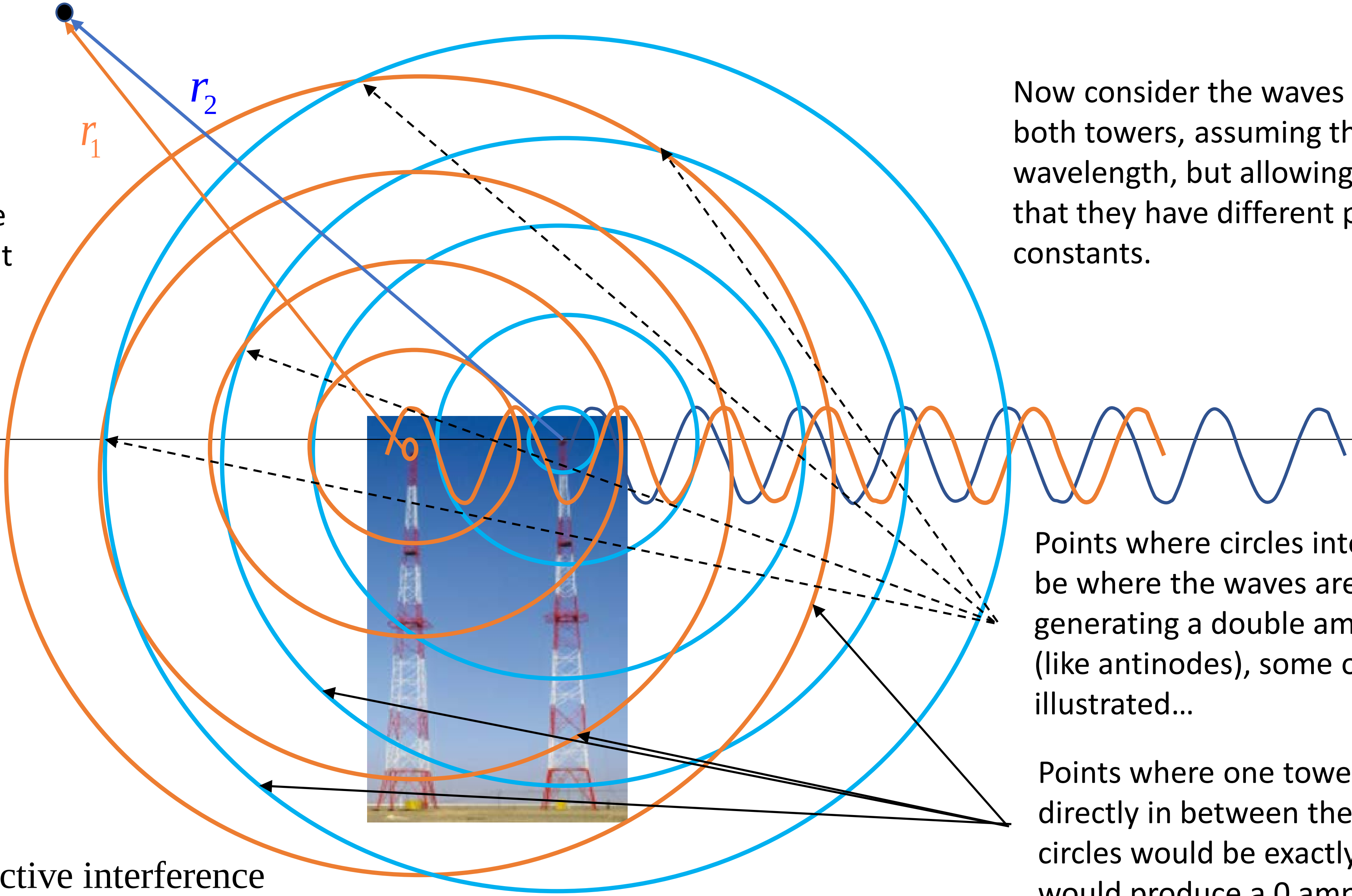
These circles would be traveling with the wave, outward away from the tower at the speed of the wave.

D.4 Multisource Interference



We want to calculate where such points lie.

Consider phase difference between the two waves at some arbitrary point.



Now consider the waves emitted by both towers, assuming they have same wavelength, but allowing for possibility that they have different phase constants.

$$\Delta\varphi = \varphi_2 - \varphi_1$$
$$= (kr_2 - \omega t + \varphi_{02}) - (kr_1 - \omega t + \varphi_{01})$$
$$= k\Delta r + \Delta\varphi_0$$

Points where circles intersect would be where the waves are in phase, generating a double amplitude oscillation (like antinodes), some of which are illustrated…

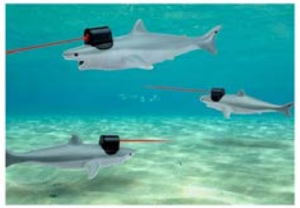
Then,

$\Delta\varphi = 2\pi m$ constructive interference

$\Delta\varphi = 2\pi m_{1/2}$ destructive interference

Points where one tower's circle lies directly in between the other tower's circles would be exactly out of phase, and would produce a 0 amplitude oscillation, like a node. For instance,

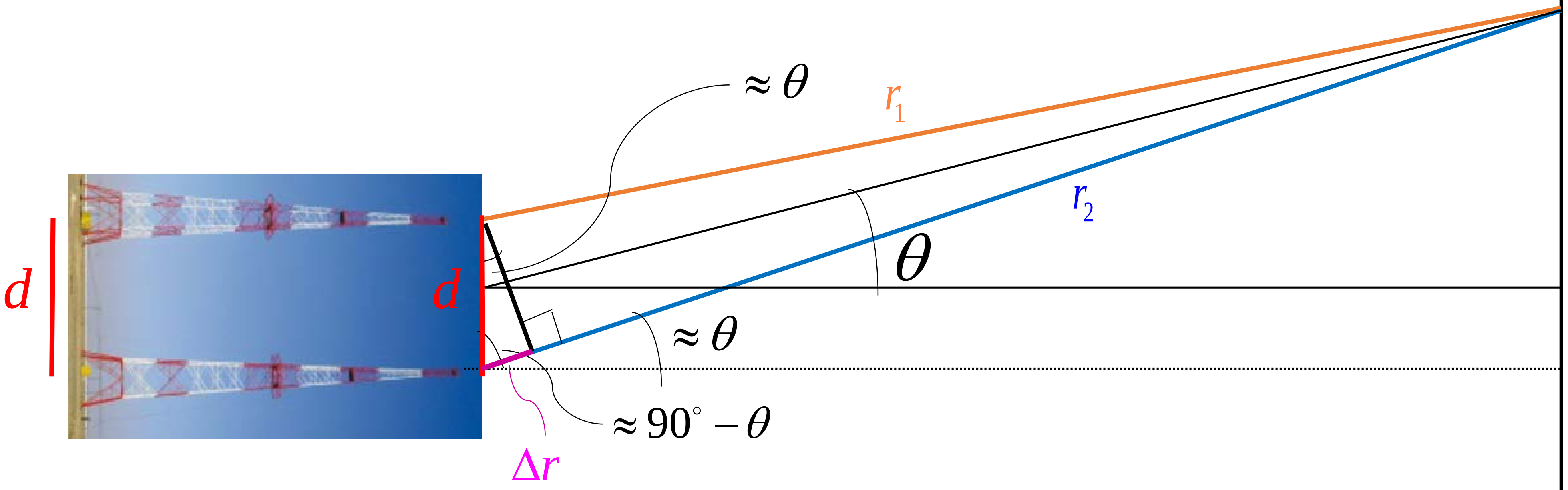
D.4 Multisource Interference

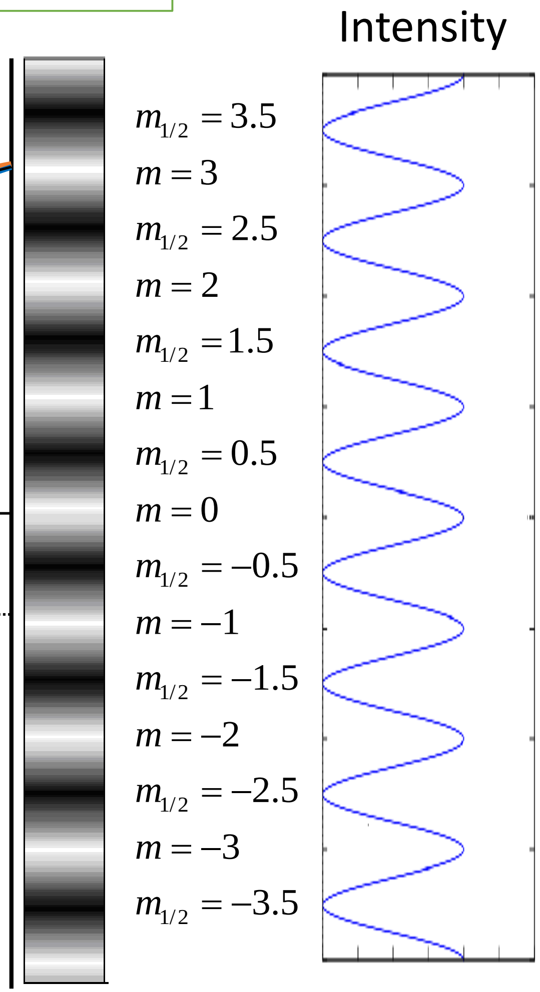


At this point, we usually make two simplifications:

(1) Assume sources are phase coherent, meaning $\Delta\phi_0 = \phi_{02} - \phi_{01} = 0$.
(2) Restrict attention to points along line perpendicular to sources.

So let's say our sources are d meters apart, and emit wavelengths λ.
At which angles, ϴ, will we get constructive or destructive interference?





$$\Delta\varphi = 2\pi m$$

$$k\Delta r = 2\pi m$$

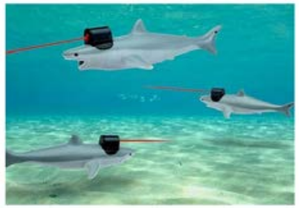
$$\left(\frac{2\pi}{\lambda}\right)\Delta r = 2\pi m$$

$$\Delta r = m\lambda$$

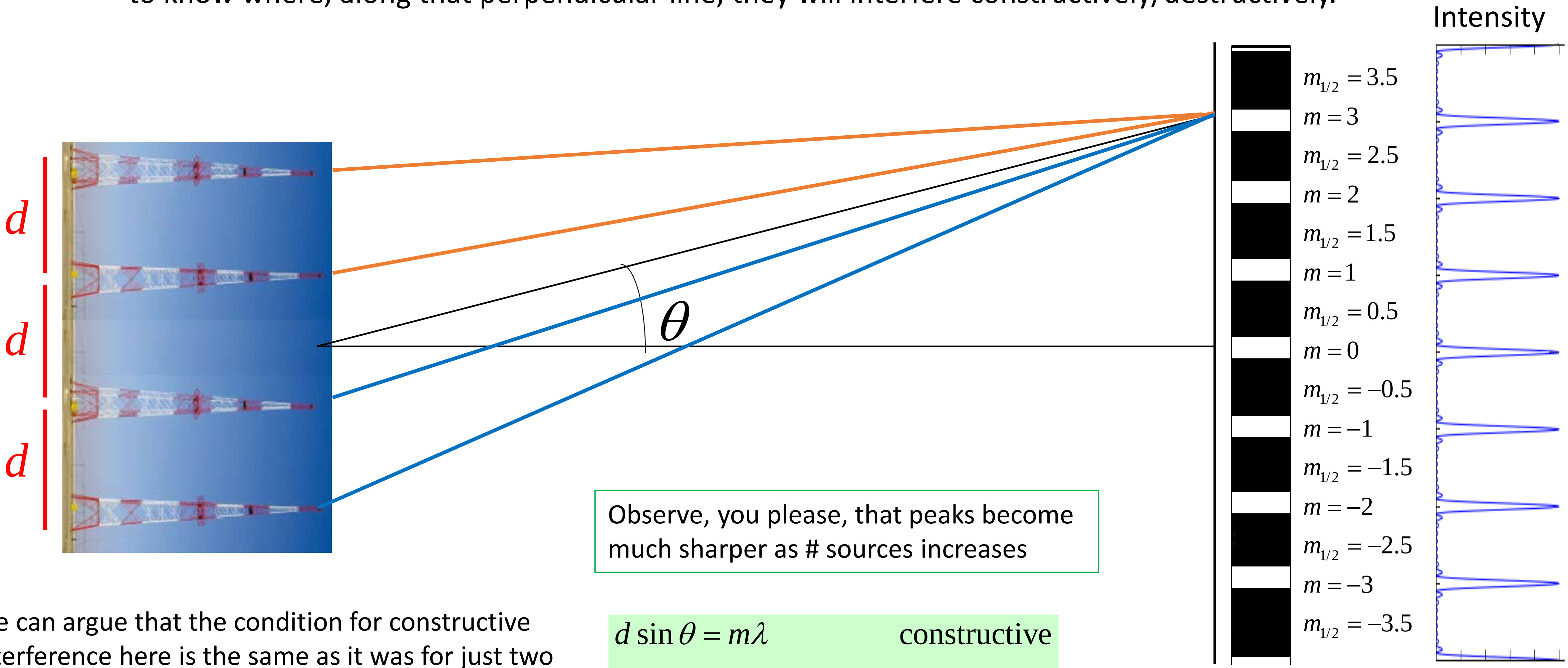
$$d\sin\theta = m\lambda$$

$d\sin\theta = m\lambda$	constructive
$d\sin\theta = m_{1/2}\lambda$	destructive

D.4 Multisource Interference



A similar case of importance is when we have more than just two sources. Again, we'd like to know where, along that perpendicular line, they will interfere constructively/destructively.



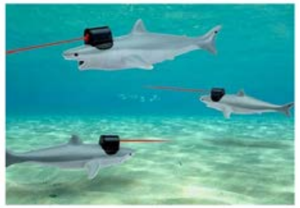
Observe, you please, that peaks become much sharper as # sources increases

We can argue that the condition for constructive interference here is the same as it was for just two sources, since if any two of the sources here are in phase, they will all be in phase. So then,

$$d \sin\theta = m\lambda \qquad \text{constructive}$$

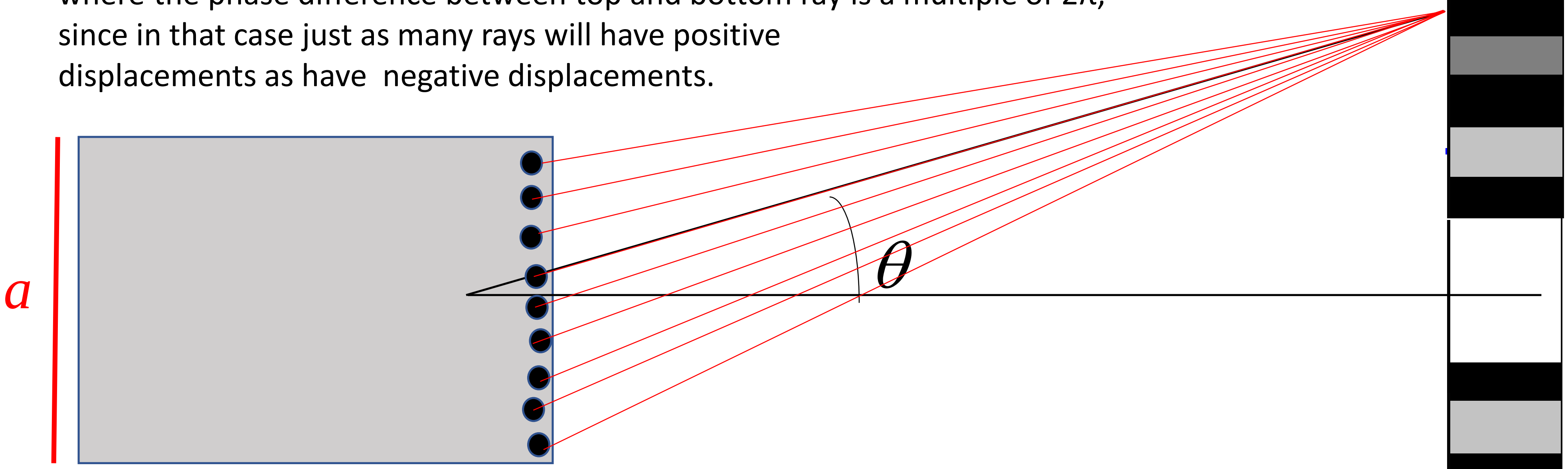
$$d \sin\theta = m_{1/2}\lambda \qquad \text{destructive}$$

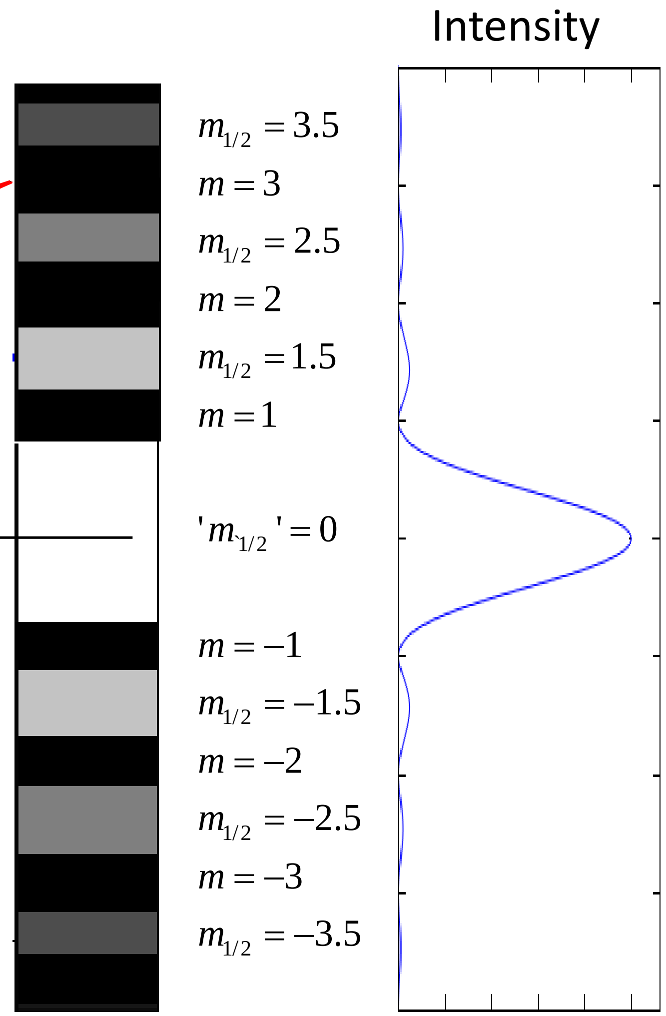
D.4 Multisource Interference

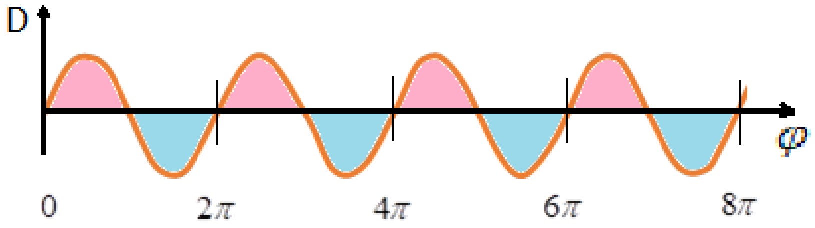


The logical limit of our discussion would be to consider an infinite array of sources. While prima facie dumb, we do actually have something like this: lasers.

In this case, its easier to locate the zero intensity spots. These will occur where the phase difference between top and bottom ray is a multiple of 2π, since in that case just as many rays will have positive displacements as have negative displacements.





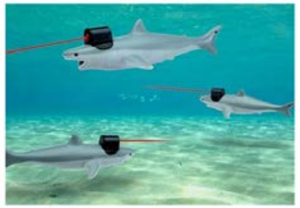


Observe, if you will, the central maximum contains by far the majority of the wave. This is how lasers stay ‘focused’.

$$\Delta\varphi_{top/bottom} = 2\pi m \rightarrow k\Delta r = 2\pi m \rightarrow \frac{2\pi}{\lambda} a\sin\theta = 2\pi m \rightarrow a\sin\theta = m\lambda$$

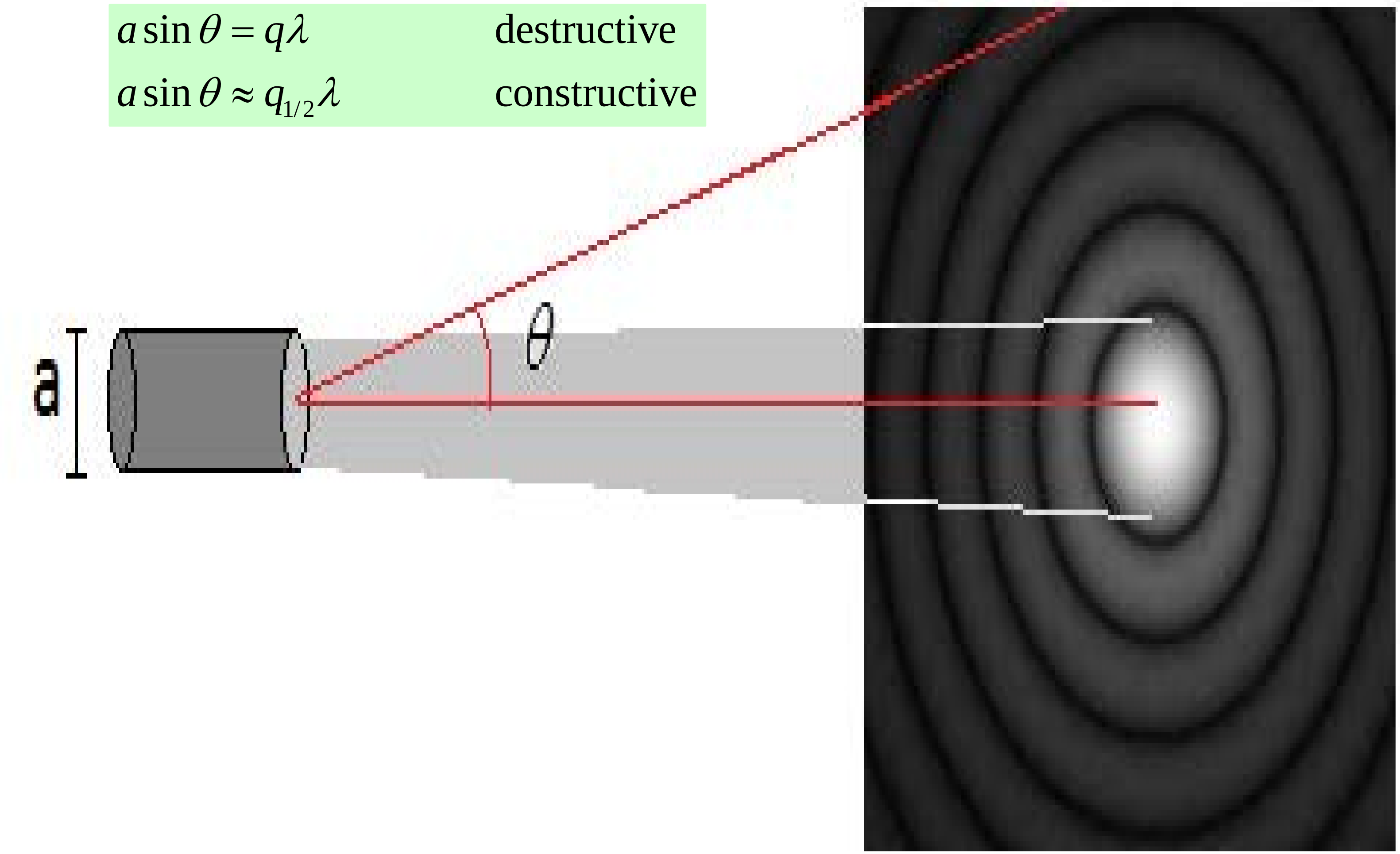
$a\sin\theta = m\lambda$	destructive
$a\sin\theta \approx m_{1/2}\lambda$	constructive

D.4 Multisource Interference

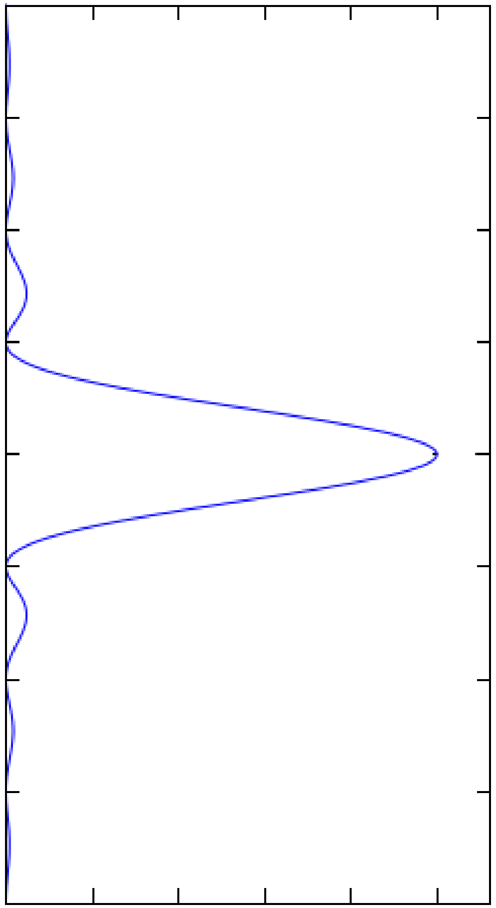


Last, most lasers have circular apertures, rather than rectangular.
In such cases, the formula for the locations of the max/zero intensities changes somewhat.
m’s are replaced with q’s. And a more sophisticated analysis would reveal:

$a\sin\theta = q\lambda$	destructive
$a\sin\theta \approx q_{1/2}\lambda$	constructive

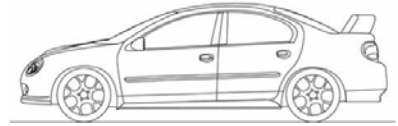


$q_{1/2} = 3.7$
$q = 3.24$
$q_{1/2} = 2.68$
$q = 2.23$
$q_{1/2} = 1.64$
$q = 1.22$
$'q_{1/2}' = 0$
$q = -1.22$
$q_{1/2} = -1.64$
$q = -2.23$
$q_{1/2} = -2.68$
$q = -3.24$
$q_{1/2} = -3.7$



D.4 Multisource Interference

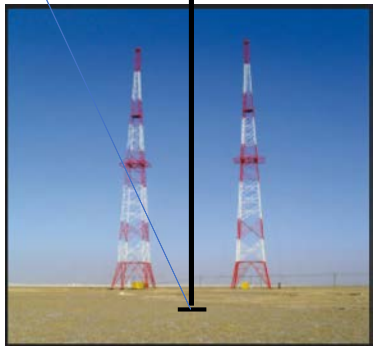
(a) You're listening to 92.1FM (i.e. f = 92.1 MHz) broadcast by two antennas a distance of 300m apart. Starting from directly in between the two antennas, how far would you drive left before you hit the first 'dead' spot, where the radio signal is weak? You can take the speed of radio waves to be c = 3×10^8m/s.



x = ?

1000m

0.31°



300m

So general formula for dead spots was: $d\sin\theta = m_{1/2}\lambda$

Therefore angle to first dead spot would be described by:

$$d\sin\theta = 0.5\lambda$$

$$\theta = \sin^{-1}\left(\frac{0.5\lambda}{d}\right)$$

$$= \sin^{-1}\left(\frac{0.5\lambda}{300}\right)$$

$$= \sin^{-1}\left(\frac{0.5\times3.26}{300}\right)$$

$$= 0.31^\circ$$

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{92.1\times10^6} = 3.26\,\text{m}$$

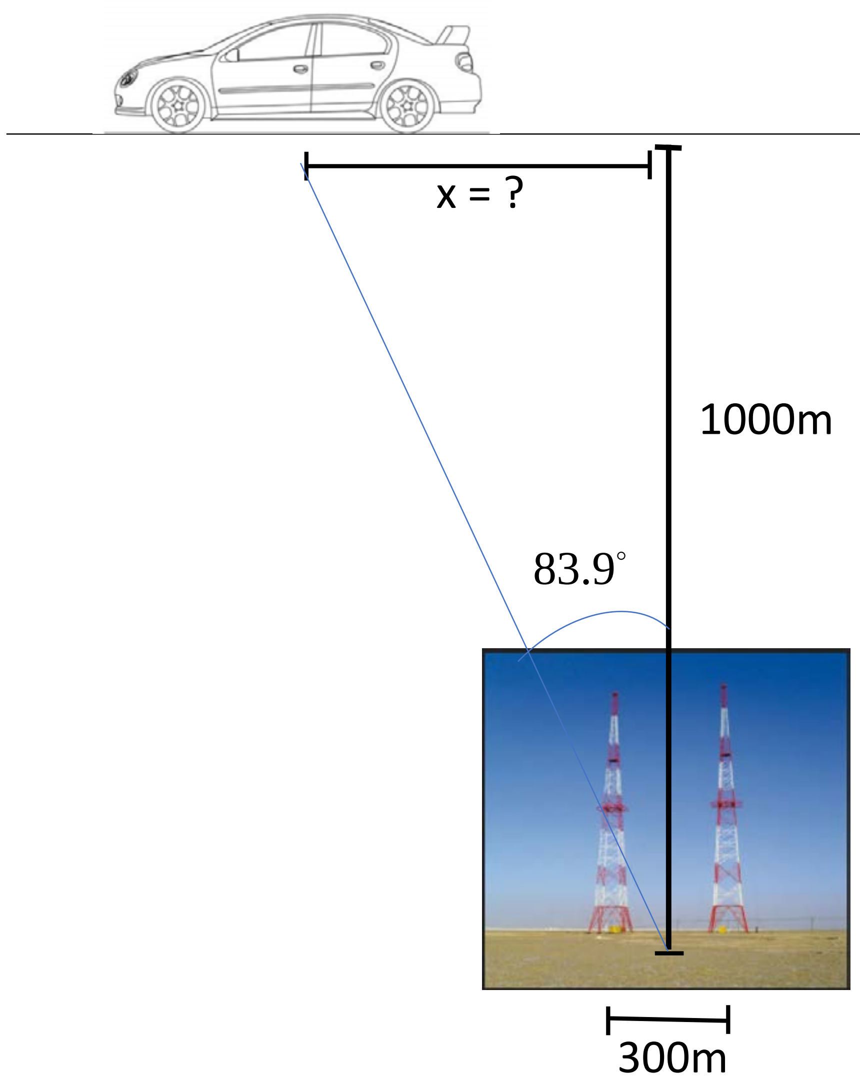
Then we can get x via:

$$\tan(0.31^\circ) = \frac{x}{1000}$$

$$x = 1000\times\tan(0.31^\circ)$$
$$= 5.4\,\text{m}$$

D.4 Multisource Interference

(b) How far would you travel before you reach the last dead spot?



To see where the last dead spot is, we need to see what the largest m would be.

$$d \sin\theta_{\max} = m_{\max}\lambda$$

$$d \sin 90^\circ = m_{\max}\lambda$$

$$m_{\max} = \frac{d}{\lambda} = \frac{300}{3.26} = 92$$

So last dead spot would correspond to $m_{1/2}$ = 91.5, and the corresponding ϴ will be:

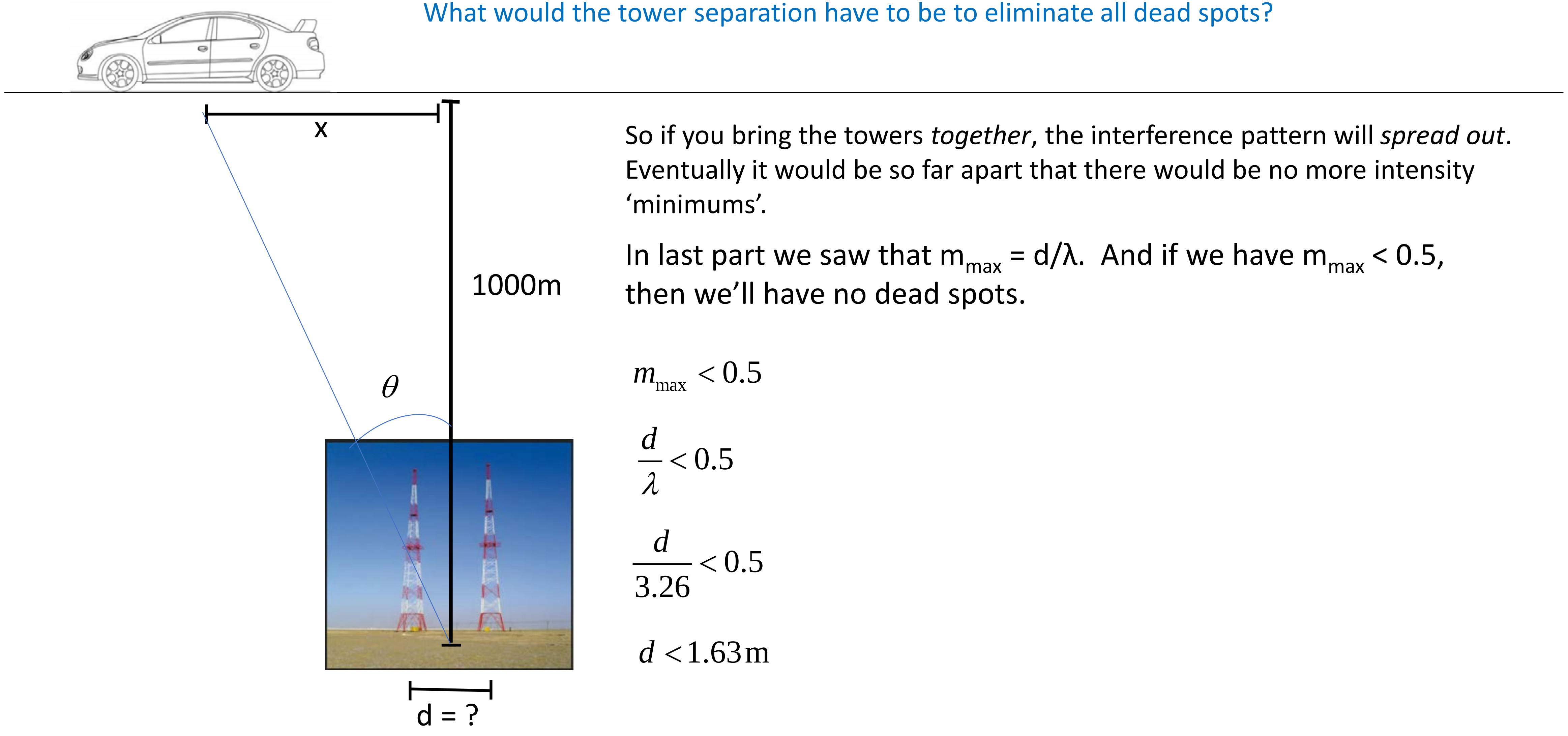
$$(300)\sin\theta = (91.5)(3.26)$$

$$\theta = \sin^{-1}\left[\frac{(91.5)(3.26)}{300}\right] = 83.9^\circ$$

And the corresponding x-coordinate would be: $x = 1000\tan 83.9^\circ = 9.36\,\mathrm{km}$

D.4 Multisource Interference

(c) Would bringing the towers closer together, or further apart, reduce the number of dead spots? What would the tower separation have to be to eliminate all dead spots?



So if you bring the towers *together*, the interference pattern will *spread out*. Eventually it would be so far apart that there would be no more intensity 'minimums'.

In last part we saw that $m_{max} = d/\lambda$. And if we have $m_{max} < 0.5$, then we'll have no dead spots.

$$m_{\max} < 0.5$$

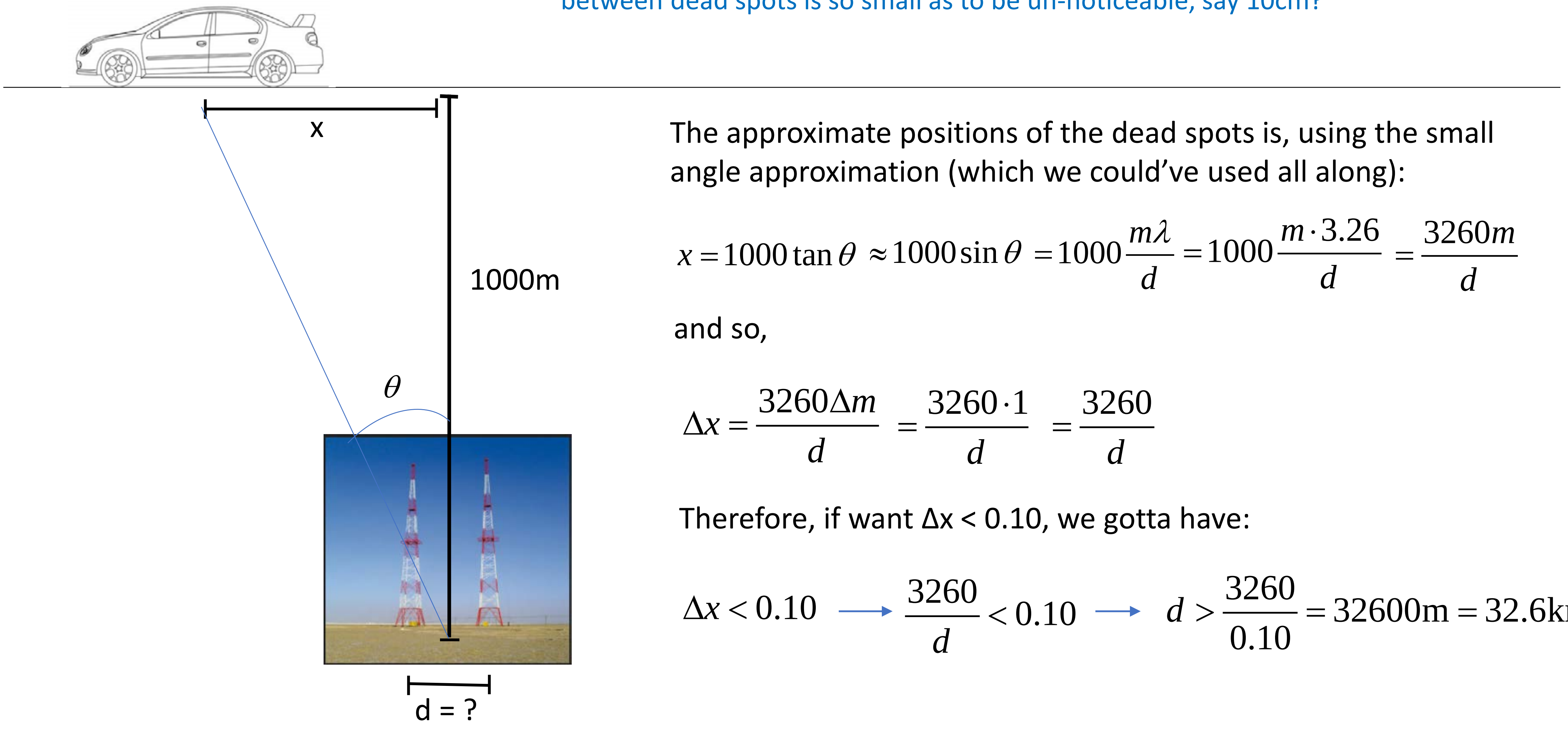
$$\frac{d}{\lambda} < 0.5$$

$$\frac{d}{3.26} < 0.5$$

$$d < 1.63\,\mathrm{m}$$

D.4 Multisource Interference

(d) On the other hand, if you spread the towers apart, the interference pattern will compress. How far apart should the towers be so that the distance between dead spots is so small as to be un-noticeable, say 10cm?



The approximate positions of the dead spots is, using the small angle approximation (which we could've used all along):

$$x = 1000\tan\theta \approx 1000\sin\theta = 1000\frac{m\lambda}{d} = 1000\frac{m\cdot 3.26}{d} = \frac{3260m}{d}$$

and so,

$$\Delta x = \frac{3260\Delta m}{d} = \frac{3260\cdot 1}{d} = \frac{3260}{d}$$

Therefore, if want Δx < 0.10, we gotta have:

$$\Delta x < 0.10 \longrightarrow \frac{3260}{d} < 0.10 \longrightarrow d > \frac{3260}{0.10} = 32600\text{m} = 32.6\text{km}$$

D.4 Multisource Interference

You're part of a special ops unit targeting Voldemort for destruction. But he's holding a unicorn hostage. So you need to carefully position your targeting laser a distance L away so that all the light contained within its second order minimum is positioned on Voldemort – to clearly distinguish to the incoming missile him and the unicorn. What's the furthest away you can be?

So the angle to the second minimum is, apparently:

$$a\sin\theta \approx q\lambda \qquad q = 2.23$$

$$(0.03)\sin\theta \approx (2.23)(12\times10^{-6})$$

$$\theta = \sin^{-1}\left(\frac{(2.23)(1200\times10^{-9})}{0.03}\right) = 0.051^{\circ}$$

And so L must given by:

$$\tan(0.051^{\circ}) = \frac{0.75/2}{L}$$

$$L = \frac{0.75/2}{\tan(0.051^{\circ})} = 421\text{m}$$

laser aperture a = 3cm